Progress Report
on ONR Grant N00014-92-J-1008
for a Period of Time October 1, 1993 - September 30, 1994

by

Michael. V. Klibanov
Principal Investigator
Department of Mathematics
University of North Carolina at Charlotte
Charlotte, NC 28223

and

Semion Gutman
Subcontractor
Department of Mathematics
University of Oklahoma
Norman, OK 73019

1. Theoretical Analysis of Numerical Schemes

Our major theoretical effort was devoted to development of convergent numerical algorithms for 3-Dimensional Inverse Scattering Problems (ISP) in strongly scattering media. Term "strongly scattering" means that coefficients of corresponding PDE can have arbitrary large variations unlike weakly scattering case in which those coefficients are close with constants. Although a large amount of publications is devoted to numerical methods for n-D ISP (n=2,3) to our knowledge, rigorous convergence analysis has not been done yet, in strongly scattering case, cf. [1]. Most of the current methods work with multi-sources ISP. That is their major mathematical requirement consists in simultaneous treatment of information obtained from many sources. These ISPs are overdetermined in 3-D case, which leads to excessive demands on computer resources. An alternative is to work with single source 3-D ISP which are exactly determined. While the latter problems are less informative, but one can improve information in a step by step procedure by solving consecutively ISPs with different source locations. In such an approach CPU time will be added rather than multiplied. Besides, the number of needed sources should be much less than in multi-sources case, which should lead to lesser experimental complexity as well.

In [2] we have developed a novel approach to convergent algorithms for single source 3-D ISPs for the hyperbolic equation

$$u_{tt}=\Delta u+b(x)u_t+a(x)u+\delta(x,t), \quad u\big|_t < 0 \equiv 0,$$

where $x\epsilon\mathbb{R}^3$ and either of coefficients a (x) or b(x) is unknown. A globally convergent algorithm. This algorithm is based on a cost functional which was proved to be uniformly strictly convex on each (reasonable) a priori given compact set. We also wish to point out that this method can work with incomplete data collections such as backscattering data, which is important for NAVY needs (Sea Ice imaging, for example).

2. Solution of a problem with real experimental data.

As we reported previously, we got involved in a collaboration with experimentalists: (i) Professor Robert F. Alfano, Institute for Ultrafast Spectroscopy and Lasers, City College of CUNY, New York; (ii) Doctor Jacob Grun, Plasma Physics Division of Naval Research Laboratory; (iii) Doctor Randall Barbour, Department of Pathology, SUNY Health Science Center at Brooklyn (NY).

Our goal is to apply our methods for 3-D images reconstruction of inclusions hidden in turbid media on the basis of light intensity measurements: light is emanated by an ultrafast laser. The basic equations for light intensity is (1) with $b < 0$. The latter inequality described losses in the media. At least two possible applications are : (a) mines detection in the coastal water, which conforms with ONR goals and (b) early breast cancer diagnosis. We obtained experimental data from Drs. Alfano and Grun. Before we proceed with programming of our sophisticated algorithm [2] we decided to see first

whether one can indeed image inclusions with a very simple a priori known geometry, such as layered media. Hence Prof. Alfano provided us with experimental data for so-called "three-layer" medium. However, we did not know where the inner layer was located neither its optical parameters. Using just a very simple inversion scheme we accurately reconstructed the image.

We believe that this solution can be applied to sea ice imaging through microwaves radiation. The point is that while the range of parameters is different, but mathematics is the same.

3. Work in progress.

We are programming the method [2]. Using this code we will get image on the basis of experimental data obtained from both places: NRL and Alfano's group. Likewise, we will also continue active contacts with Barbour's group trying to compare and possibly combine our two approaches to the imaging challenge.

4. Publications [2-7] In particular, the first computational result for a full 3-D ISP is presented in [6].

5. Participation in the conferences.

a. Advances in Optical Imaging and Photon Migration, Orlando, FL, March 1994.

b. Optical Engineering and Aerospace Sensing, Orlando, FL, April 1994.

c. Inverse Problems in Engineering Sciences (Klibanov was an invited speaker), Osaka, Japan, July 1994.

References

1. D. Colton and R. Kress, Inverse Acoustic and Electromagnetic Scattering Theory, Appl. Math. Sci., V. 93, Springer-Verlag, New York, 1992.

2. S. Gutman, M.V. Klibanov and A. V. Tikhonravov, convexity for 3-Dimensional inverse scattering problem, submitted to Inverse Problems.

3. S. Gutman and M.V. Klibanov, Iterative method for multi-dimensional inverse scattering problems of fixed freuencies, Inverse Problems 10 (1994), 573-599.

4. M.V. Klibanov and P. Sacks, Use of partial knowledge of the potential in the phase problem of inverse scattering, Journal of Computational Physics, 112 (1994), 273-281.

5. M. V. Klibanov and O.V. Ioussoupova, Uniform strict convexity of a cost functional for 3-dimensional inverse scattering problem, to appear in SIAM J. Math. Anal., 1995, January issue.

6. S. Gutman and M.V. Klibanov, Three-dimensional inhomogeneous media imaging, to appear in Inverse Problems, 1994.

7. M. Kazemi and M.V. Klibanov, Stability estimates for ill-posed Cauchy problems involving huperbolic equations and inequalities, Applicable Analysis, 50 (1993), 93-102